Radicali. Proprietati

1. $\sqrt[m]{a} = a^{\frac{1}{m}}, (a > 0);$
2. $\sqrt[m]{\frac{1}{a}} = \frac{1}{\sqrt[m]{a}} = a^{-\frac{1}{m}}, (a > 0);$
3. $(\sqrt[m]{a})^m = a, (a \geq 0);$
4. $\sqrt[m]{a} \cdot \sqrt[m]{b} = \sqrt[m]{a \cdot b}, (a, b \geq 0);$
5. $\left(\sqrt[m]{\frac{1}{a}}\right)^m = \frac{1}{a}, (a > 0);$
6. $\sqrt[m]{a} \cdot \sqrt[m]{b} \cdot \sqrt[m]{c} = \sqrt[m]{a \cdot b \cdot c}, (a, b, c \geq 0);$
7. $\sqrt[m]{a} : \sqrt[m]{b} = \sqrt[m]{\frac{a}{b}}, (a \geq 0, b > 0)$
8. $\sqrt[m]{a} \cdot \sqrt[n]{a} = \sqrt[mn]{a^{m+n}}$
9. $\sqrt[m]{a} : \sqrt[n]{a} = \sqrt[mn]{a^{n-m}}, (a > 0);$
10. $\sqrt[n]{a^{mn}} = a^m, (a \geq 0);$
11. $\sqrt[m]{a^n} = (\sqrt[m]{a})^n = a^{\frac{n}{m}}, (a \geq 0);$
12. $\sqrt[mn]{a^{mp}} = \sqrt[n]{a^p}, a > 0);$
13. $\sqrt[m]{a^p} \cdot \sqrt[n]{b^q} = \sqrt[mn]{a^{pn} \cdot b^{qm}}, (a, b \geq 0);$
14. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}, (a \geq 0);$
15. $\sqrt[m]{a^p} : \sqrt[n]{b^q} = \sqrt[mn]{a^{pn} : b^{qm}}, (a \geq, b > 0);$
16. $\sqrt{a^2} = |a|, (a \in \mathbb{R});$

Identitati fundamentale

Oricare ar fi $x, y, z, t, a, c, d \in \mathbb{R}$ si $n \in \mathbb{N}$, avem:

$1.a^2-b^2=(a-b)(a+b);4ab=(a+b)^2-(a-b)^2;$

$2.(a^2+b^2)(x^2+y^2)=(ax-by)^2+(ay+bx)^2;$

$3.(a^2+b^2+c^2+d^2)(x^2+y^2+z^2+t^2)=$

$=(ax-by-cz-dt)^2+(bx+ay-dz-ct)^2+$

$+(cx+dy+az-bt)^2+(dx-cy+bz+at)^2;$

$4.a^3-b^3=(a-b)(a^2+ab+b^2);$

$5.a^3+b^3=(a+b)(a^2-ab+b^2);$

$6.x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx);$

$7.x^3+y^3+z^3=(x+y+z)^3-3(x+y)(y+z)(z+x);$

$8.a^4-b^4=(a-b(a+b)(a^2+b^2);$

$9.a^4+b^4=(a^2+b^2-ab\sqrt{2})(a^2+b^2+ab\sqrt{2});$

$10.a^5-b^5=(a-b)(a^4+a^3b+a^2b^2+ab^3+b^4);$

$11.a^5+b^5=(a+b)(a^4-a^3b+a^2b^2-ab^3+b^4);$

$12.(1+a)(1+a^2+a^4)=1+a+a^2+a^3+a^4+a^5;$

$13.a^6+b^6=(a^3-2ab^2)^2+(b^3-2a^2b)^2$

$14.a^n-b^n=(a-b)(a^{n-1}+a^{n-2}b+\ldots+ab^{n-2}+b^{n-1});$

$15.a^{2n}-b^{2n}=(a^2-b^2)(a^{2n-2}+a^{2n-4}b^2+\ldots+a^2b^{2n-4}+b^{2n-2});$

$16.a^{2n+2}+b^{2n+1}=(a+b)(a^{2n}+a^{2n-1}b+\ldots+ab^{2n-1}+b^{2n});$

$17.(1+a+a^2+\ldots+a^n)(1+a^{n+1})=1+a+a^2+\ldots+a^{2n+1};$

Modulul unui numar real

$$|x|=\begin{cases}-x, daca & x<0\\ 0, daca & x=0\\ x, daca & x>0\end{cases}$$

proprietati : $\forall\ x, y \in \mathbb{R}$ *avem*:

1. $|x| = 0 \Leftrightarrow x = 0;$
2. $|-x| = |x|;$
3. $|x| = |y| \Leftrightarrow x = y$ *sau* $x = -y;$
4. $|x| = a \Leftrightarrow -a = x = a,$ *cu* $a \in \mathbb{R}$
5. $|-x| \le x \le |x|;$
6. $|x + y| \le |x| + |y|;$
7. $|x - y| \le |x| + |y|;$
8. $\big||x| - |y|\big| \le |x - y|;$
9. $\big||x| - |y|\big| \le |x + y| \le |x| + |y|;$
10. $|xy| = |x| \cdot |y|;$
11. $\left|\frac{x}{y}\right| = \frac{|x|}{|y|},\ (y \neq 0);$

Extragerea radacinii de ordinul n dintr-un numar complex

$$(\sqrt[n]{z})_k = r^{\frac{1}{n}}\left[\cos\frac{u + 2k\pi}{n} + i\sin\frac{u + 2k\pi}{n}\right],\ k = 0,1,2,\ldots,n-1;$$

$$(\sqrt[n]{1})_k = \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n},\ k = 0,1,2,\ldots,n-1;$$

$$(\sqrt[n]{-1})_k = \cos\frac{(2k+1)\pi}{n} + i\sin\frac{(2k+1)\pi}{n},\ k = 0,1,2,\ldots,n-1;$$

Ecuatii si inecuatii de gradul al II-lea

Ecuatii de gradul al doilea

$$ax^2 + bx + c = 0,\ a, b, c \in \mathbb{R}\ ,\ a \neq 0$$

Rezolvare:

$$x_1 = \frac{-b+\sqrt{\Delta}}{2a} \; ; \; x_2 = \frac{-b-\sqrt{\Delta}}{2a} \; ; \; \Delta = b^2 - 4ac \; sau$$

$$x_1 = \frac{-b`+\sqrt{\Delta`}}{a} \; ; \; x_2 = \frac{-b`+\sqrt{\Delta`}}{a} \; ; \; b = 2b` \; ; \; \Delta` = b`^2 - 4ac.$$

$$Viete: \quad x_1 + x_2 = -\frac{b}{a} \; ; x_1 x_2 = \frac{c}{a}$$

Studiul ecuatiei de gradul al doilea

$$x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2 = S^2 - 2P$$
$$x_1^3 + x_2^3 = (x_1 + x_2)^3 - 3x_1x_2(x_1 + x_2)S^3 - 2SP$$
$$x_1^4 + x_2^4 = (x_1 + x_2)^4 - 2x_1^2x_2^2 = S^4 - 4S^2P + 2P^2$$

Inecuatii fundamentale de gradul al II-lea

$1.ax^2 + bx + c > 0, \quad a,b,c \in \mathbb{R}, \quad a \neq 0, \quad S = multimea \;\; solutiilor$

Δ	a	S
$\Delta > 0$	$a > 0$	$(-\infty, x_1) \cup (x_2 + \infty)$
$\Delta > 0$	$a < 0$	(x_1, x_2)
$\Delta = 0$	$a > 0$	$\mathbb{R} \setminus \{x_1\}$
$\Delta = 0$	$a < 0$	$\varnothing$
$\Delta < 0$	$a > 0$	$\mathbb{R}$
$\Delta < 0$	$a < 0$	$\varnothing$

$2.ax^2 + bx + c \geq 0 \;\; , \;\; a,b,c \in \mathbb{R} \;\; , a \neq 0 \;\; , S = multimea \;\; solutiilor$

Δ	a	S
$\Delta > 0$	$a > 0$	$(-\infty, x_1) \cup (x_2 + \infty)$
$\Delta > 0$	$a < 0$	$[x_1, x_2]$
$\Delta = 0$	$a > 0$	$\mathbb{R}$
$\Delta = 0$	$a < 0$	$\{x_1\}$
$\Delta < 0$	$a > 0$	$\mathbb{R}$
$\Delta < 0$	$a < 0$	$\varnothing$

Logaritmi Proprietatii

1. $\log_a b = \log_a c \Leftrightarrow b = c,\ (b,c>0);$
2. $\log_a a = 1;$
3. $\log_a 1 = 0;$
4. $\log_a a^c = c;\ \log_a \frac{1}{b} = -\log_a b;$
5. $\log_a b \cdot \log_b a = 1;$
6. $\log_a \sqrt[m]{b} = \frac{1}{m}\log_a b,\quad (b>0, m\in\mathbb{N}, m\geq 2);$
7. *Schimbarea bazei* $\log$.

$$\log_a b = \frac{\log_c b}{\log_c a};$$

8. $x>0\ \ si\ \ y>0 \Rightarrow \log_a x + \log_a y;$
9. $x>0\ \ si\ \ y>0 \Rightarrow \log_a \frac{x}{y} = \log_a x - \log_a x - \log_a y;$
10. $a>1\ \ si\ \ x\in(0,1) \Rightarrow \log_a x<0;\quad a>1\ \ si\ \ x>1 \Rightarrow \log_a x>0;$
11. $0<a<1\ \ si\ \ x\in(0,1) \Rightarrow \log_a x>0;\quad 0<a<1\quad x>1 \Rightarrow \log_a x<0$
12. $a>1\ \ si\ \ 0<x<y = \log_a x < \log_a y;\ \ 0<a<1\ \ si\ \ 0<x<y = \log_a x > \log_a y;$
13. $x>0, y>0, a>0, b>0,\ \ a\neq 1, b\neq 1 \Rightarrow \frac{\log_a x}{\log_a y} = \frac{\log_b x}{\log_a y};$
14. $x>0, a>0,\ \ a\neq 1,\ \ n\in\mathbb{N}\ \ \Rightarrow \log_a x = \log_{a^n} x^n\ ;$
15. $x\in\mathbb{R},\ \ a>0,\ \ a\neq 1 \Rightarrow a^x = e^{x\ln a};$

Combinari Proprietatii

$$1.C_n^1 = n;\ C_n^n = C_n^0 = C_0^0 = 1;$$
$$2.C_n^n = C_n^{n-m};C_n^m = C_{n-1}^m + C_{n-1}^{m-1};$$
$$3.C_n^m = C_n^{m-1} + C_{n-2}^{m-1} + \ldots + C_{n+1}^{m-1} + C_n^{m-1} + C_{n-1}^{m-1};$$
$$4.\frac{n!}{p_1!p_2!\ldots p_3!} = C_n^{p_1} \cdot C_{n-p_1}^{p_2} \ldots C_{n-(p_1+\ldots+p_{m-1})} \text{ unde } p_1 + \ldots + p_{m-1} < n;$$

Progresii aritmetice

Se numeste progresie aritmeticaun sir de numere $a_1,a_2,a_3,\ldots,a_n,\ldots$in care fiecare termen, incepand cu a_2, se obtine din cel precedent prin adaugarea unui numar constant numit ratia progresiei. Se noteaza $a_1,a_2,a_3,\ldots a_n,\ldots$

Daca a_1 este primul termen, an cel de-al n-lea termen, r ratia, n numarul termenilor si S_n suma celor n termeni, atunci avem:

$$a_n = a_{n-1} + r, \quad n \geq 2$$
$$a_n = a_1 + (n-1)r, \quad n \geq 2$$
$$S_n = a_1 + a_2 + \ldots + a_n, \quad S_n = \frac{(a_1 + a_n)n}{2}$$
$$S_n = \frac{2a_1 + (n-1)r}{2}n$$

Matrici

Fie M={1,2,...,m} si N ={1,2,...,n}.

A: MxN —> **C ,** A(i, j)=a_{ij} se numeste matrice de tipul (m, n); cu m linii si n coloane:

$$a = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots\ldots\ldots & & \ldots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{pmatrix}$$

Notam cu $M_{m,n}$(**C**) multimea matricelor de tipul (m,n) cu elemente numere complexe.

Daca m=n atunci matricea se numeste patratica de ordinul n, iar multimea lor se noteaza $M_n(\mathbf{C})$.

Doua matrici A,B care apartin lui $M_{m,n}(\mathbf{C})$ sunt egale daca si numai daca a_y ,b_y ,oricare ar fi (i, j) apartin lui M x N.

Adunarea matricilor

$$Fie\ \ A,B\in M_{m,n}(\mathbb{C})\ \ atunci\ \ C=A+B\in M_{m,n}(\mathbb{C}),\ \ unde$$
$$c_{ij}=a_{ij}+b_{ij},\ \ \forall(i,j)\in M\times N\ \ este\ \ suma\ \ lor$$

Proprietati, oricare ar fi A,B,C care apartin lui $M_{m,n}(\mathbf{C})$:

$$1.A+B=B+A\ \ (comutativitate);$$
$$2.(A+B)+C=A+(B+C)\ \ (asociativitate);$$
$$3.A+O=O+A\ \ (elementul\ \ neutru\ \ esete\ \ matricea\ \ mula\ \ O);$$
$$4.A+(-A)=(-A)+A=O(opusul\ \ lui\ \ A\ \ este\ \ -A).$$

Inmultirea cu scalari

$$Fie\ \ A\in M_{m,n}(\mathbb{C})\ \ si\ \ \lambda\in\mathbb{C}\ \ atunci\ \ B=\lambda\cdot A\in M_{m,n}(\mathbb{C}),$$
$$unde\ \ b_{ij}=\lambda a_{ij}\ \ \forall(i,j)\in M\times N\ \ este\ \ produsul\ \ matricei\ \ A$$
$$cu\ \ scalarul\ \ \lambda.$$

Proprietati:

$$1.1\cdot A=A;$$
$$2.\lambda\cdot A=a\cdot\lambda;$$
$$3.\lambda(A+B)=\lambda A+\lambda B;$$
$$4.(\lambda+\mu)A=\lambda A+\mu A;$$
$$5.\lambda(\mu A)=(\lambda\mu)A=\mu(\lambda A);$$

Transpusa unei matrici:

$$Fie\ \ A\in M_{m,n}(\mathbb{C})\ \ atunci\ \ {}^tA\in M_{m,n}(\mathbb{C})$$
$$unde\in M_{m,n}(\mathbb{C})\ \ {}^ta_{ij}=a_{ij},\ \ \forall(i,j)\in M\times N$$

Inmultirea matricelor

Fie $A \in M_{m,n}(\mathbb{C})$ *si* $B \in M_{n,p}(\mathbb{C})$ *atunci*

$C = A \cdot B \in M_{m,p}(\mathbb{C})$, *unde* $c_{ij} = \sum_{k=1}^{n} a_{ij} b_{kj}$, $\forall (i,j) \in M \times N$

este produsul lor.

Proprietati:

1. $(A \cdot B) \cdot C = A \cdot (B \cdot C)$ (*asociativitate*);
2. $A \cdot I_n = I_n \cdot A$ (*element neutru matricea unitate*);

$$I_n = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{pmatrix} \text{ in } M_n(\mathbb{C});$$

3. $(A+B) \cdot C = A \cdot C + B \cdot C$;
4. $A \cdot (B+C) = A \cdot B + A \cdot c$.

Determinanti

Determinanti :0

Fie $M_n(\mathbb{C})$ *multimea matricilor patrate de ordin n cu elemente din* $\mathbb{C}$:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots\dots\dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}, \quad A \in M_n(\mathbb{C});$$

Se numeste determinantul matricei A, numarul

$$\det A = \sum_{\sigma \in S_n} \varepsilon(\sigma) a_{1\sigma(1)} a_{2\sigma(2)} \dots a_{n\sigma(n)}$$

$$\det A = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots\dots\dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

$\det A = a_{i1}A_{i1} + a_{i2}A_{i2} + \ldots + a_{in}A_{in}$, *unde* A_{ij} *este complementul ALGEBRIC al elementului* a_{ij} *din matricea* A:

$$A_{ij} = (-1)^{i+j} \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1\,j-1} & a_{1\,j+1} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2\,j-1} & a_{2\,j+1} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ a_{i-11} & a_{i-12} & \ldots & a_{i-1\,j-1} & a_{i-1\,j+1} & \ldots & a_{i-1n} \\ a_{i+11} & a_{i+12} & \ldots & a_{i+1\,j-1} & a_{i+1\,j+1} & \ldots & a_{i+1n} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nj-1} & a_{nj+1} & \ldots & a_{nm} \end{vmatrix}$$

Daca $C = A \cdot B$, *atunci* $\det C = \det A \cdot \det B$ $(A, B, C \in M_n(\mathbb{C}))$.